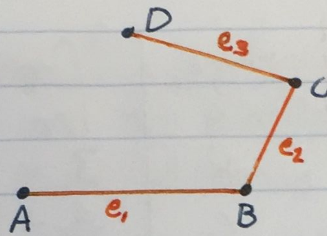
5/3/16

Chapter 10: Graphs (Deviates from textbook)

Graphs

* <u>Graph (non-directed)</u> example:

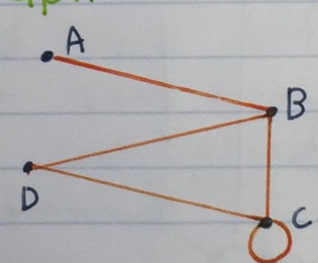


Verticies = {A, B, C, D}
Edges = {AB, BC, CD}

* <u>Adjacent</u> = If two vertices are connected by an edge, they are said to be adjacent to eachother.
  - An edge that is incedent on a vertex means that the edge is attatched to that vertex.

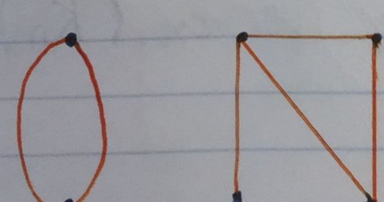
* <u>Types of Graphs</u>:

1.) Connected Graph



* <u>Bridge</u> = An edge whose remo
turns a connected graph int
a disconnected graph

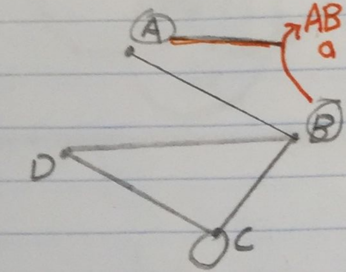
2.) Disconnected Graph



$\underbrace{\qquad\qquad\qquad}_{\text{this is the whole graph}}$

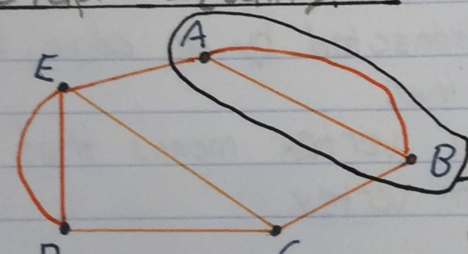
*Degree of a Vertex: The degree of a vertex = to the # of edges that are incident on the vertex.
in other words, loops count twice!

ex.)



Vertex	Degree
A	1
B	3
C	4
D	2

*Graph Equality



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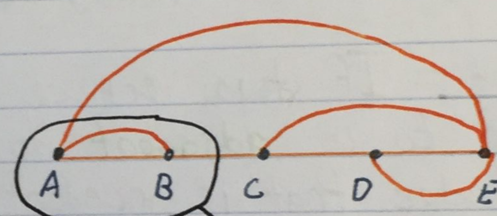
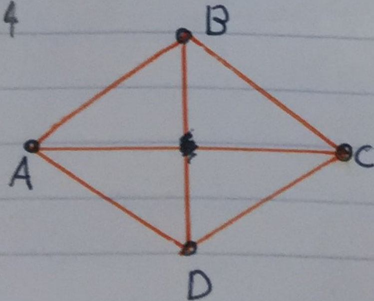


Called redundent

Vertex	Degree
A	3
B	
C	
D	
E	4

3.) Complete Graph = A graph where each vertex is adjacent to every other vertex (K_n)

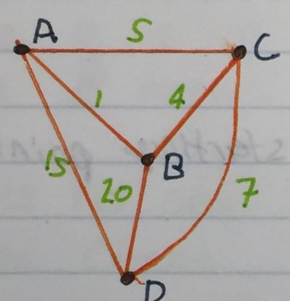
ex. K_4



4.) Weighted Graphs: The edge values will have numerical values that will be the "weight" of the edge.

What does this "weight" represent?
Depends on the Problem!
- miles / distance
- time
- cost

ex.



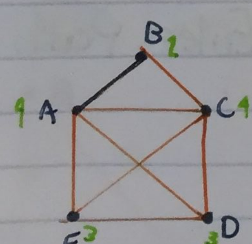
Circuits / Paths (Euler)

* Euler Circuit: You leave from a vertex, travel each edge exactly once and return to where you started.

Euler Path: You leave from a vertex, travel each edge exactly once and end up somewhere else

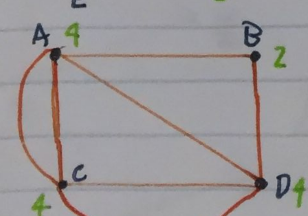
You cannot have both!!!

ex.)



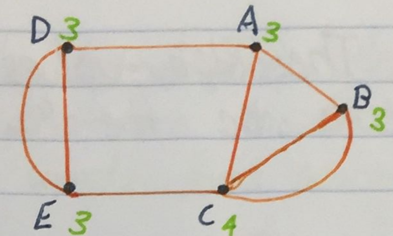
No Euler Circuit!
Yes Euler path, must start at E or D

ex.)



Euler Circuit
no Euler path

ex.)



Neither Euler Circuit or Euler Path!

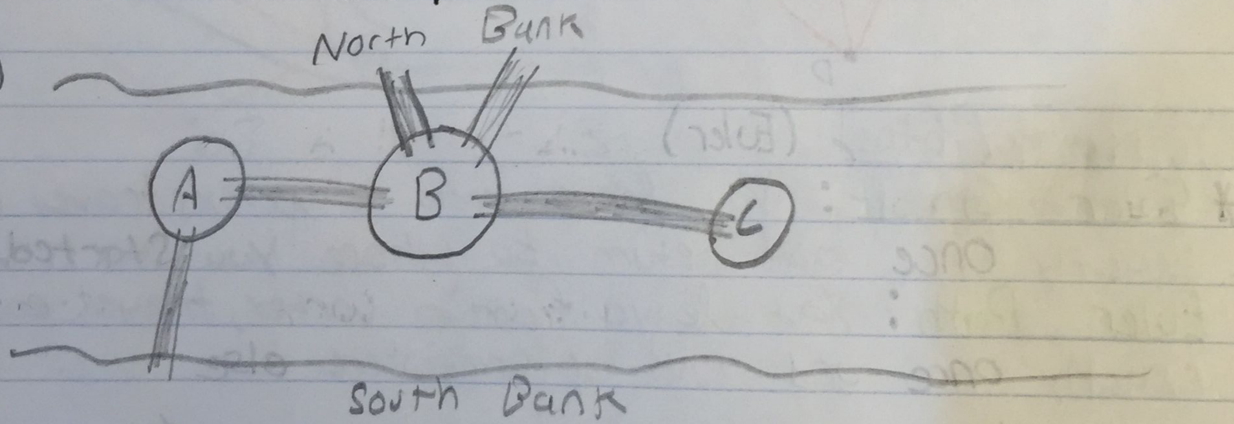
look @ the degrees of these graphs !!!!

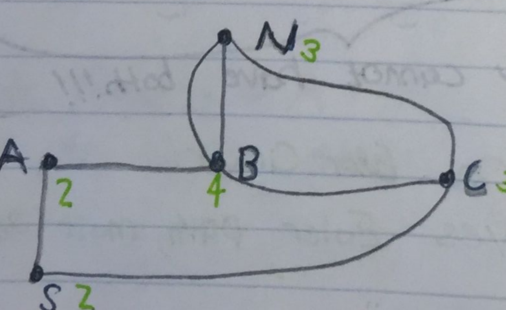
- To have an Euler Circuit a graph must be connected and each vertex must have an even degree
- To have an Euler Path a graph must be connected and exactly two of the vertices must have an odd degree. (others even)

↓

one odd degree will be the starting point & the other the end point

ex.)





Euler Path

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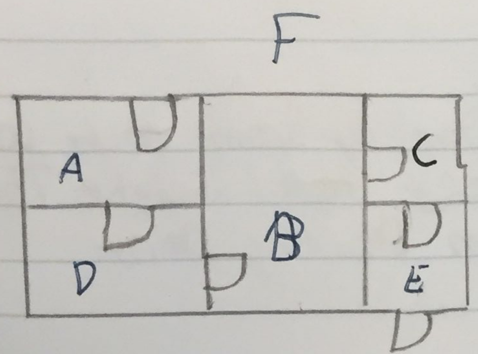
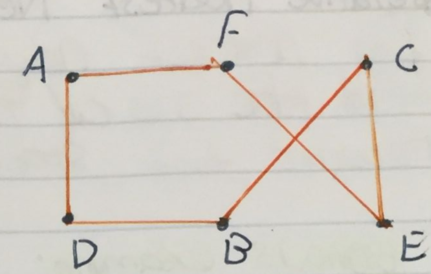




Euler Circuit

~~Hamilton Circuits~~

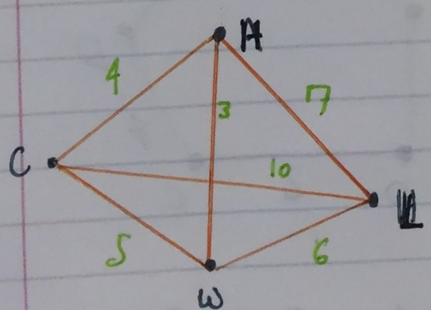
* You leave a vertex, travel to every other vertex exactly one time & return to the Starting Vertex.

There is no easy way to determine if a graph has a hamilton Circuit.

~~Traveling~~ Traveling - Salesperson Problem

* Find the most efficient way to travel. They must be weighted graphs.
* Brute Force Method: The only way to find the optimal Solution. It requires that you find the weight of each & every route.

So how many possible roots are there?



3! roots = 6 but some are mirror images of eachother.

So $\frac{6}{2} = 3$ distinct routes

Nearest Neighbor

A W C L A

$3 + 5 + 10 + 7 = 25$

just keep going to the nearest neighbor w/o repetition

Repetative nearest Neighboor

Do nearest neighboor Starting from each vertex find the one w/ the lowest total weight. Then rewrite that starting from starting point

for previous example:

H W C L H 25

L W H C L
6 3 4 10 = 23

W H C L
3 4 10 6 = 23

C H W L C ⟶ So just rewrite
4 3 6 10 = 23

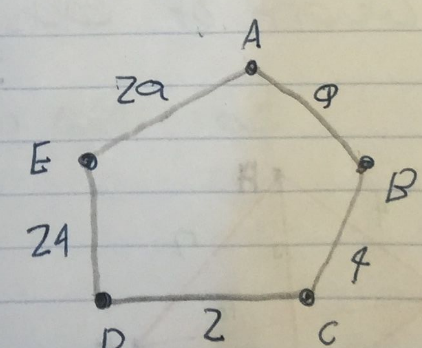
↳ H W L C H

Cheapest link

If given graph you may want to write weights in order.

ex.)

	A	B	C	D	E
A	–	9	~~7~~	~~19~~	29
B	9	–	4	~~6~~	~~14~~
C	7	4	–	2	~~11~~
D	19	6	2	–	29
E	29	14	11		–



HW = DO REVIEW