4/28/16

Y

(9.3) Counting Elements of Disjoint Sets

* The Addition Rule: let $A_1, A_2, \ldots A_k$ be mutually disjoint & $\bigcup_{n=1}^{k} A_n = A$

$$\boxed{N(A) = N(A_1) + N(A_2) + \ldots N(A_k)}$$

Pg 549 8.) 3-5 characters in length

26 upper-case
10 digits
+14 symbols
50

a.) 3 char pw: $\underline{50} \times \underline{50} \times \underline{50} = 50^3$

repetition → 4 char pw: $\underline{50} \times \underline{50} \times \underline{50} \times \underline{50} = 50^4$

5 char pw: $\underline{50} \times \underline{50} \times \underline{50} \times \underline{50} \times \underline{50} = 50^5$

total # possibilities = $50^3 + 50^4 + 50^5 = 318,875,000$

b.) 3 char pw: $\underline{50} \times \underline{49} \times \underline{48}$

no repetition → 4 char pw: $\underline{50} \times \underline{49} \times \underline{48} \times \underline{47}$

5 char pw: $\underline{50} \times \underline{49} \times \underline{48} \times \underline{47} \times \underline{46}$

total # possibilities = 259,896,000

* The Difference Rule: let $B \subseteq A \ldots$

$$\boxed{N(A-B) = N(A) - N(B)}$$

c.) 318,875,000 - 259,896,000
= 58,979,000

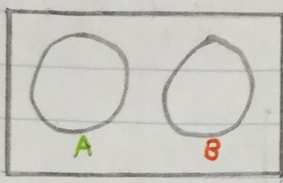
d.) $\frac{58,979,000}{318,875,000}$

12.) T-H-E-O-R-Y

a.) $\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 6!$

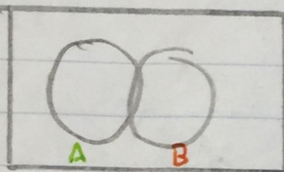
FH b.) $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \cdot 2 = 240$

* The Addition Rule for Non-Disjoint Sets:

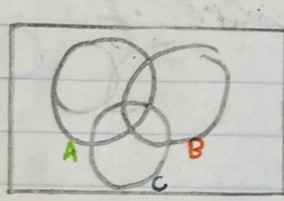


$\rightarrow$ Disjoint so $N(A \cup B) = N(A) + N(B)$

BUT...



$\boxed{N(A \cup B) = N(A) + N(B) - N(A \cap B)}$

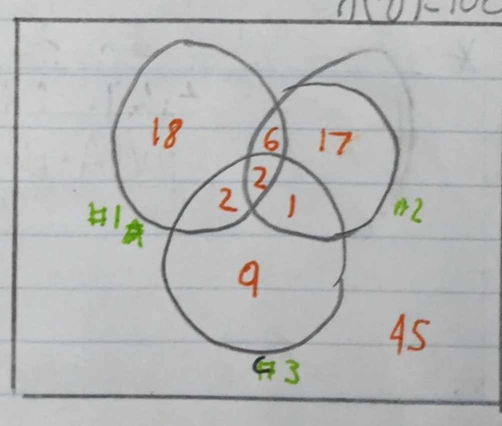


$$\boxed{N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)}$$

552 33.) $N(U) = 100$

28	✓ #1
26	✓ #2
14	✓ #3
8	✓ #1 & 2
4	✓ #1 & 3
3	✓ #2 & 3
2	✓ all 3

$n(U) = 100$



a.) 55
b.) 45
c.)
d.) 6
e.) 1
f.) 17

HW: Pg 549 4, 6, 7, 11, 39

9.5) $\boxed{\text{Combinations}}$

* Recall:

Permutations → Order matters

Combinations → order doesn't matter

$$\boxed{\binom{n}{r} = \frac{n!}{(n-r)!\,r!}}$$

7.) a.) $\binom{13}{7} = \dfrac{13!}{(13-7)!\,7!} = \dfrac{13 \cdot 12 \cdot 11 \cdot \cancel{10} \cdot \cancel{9}^{3} \cdot \cancel{8}^{4} \cdot \cancel{7!}}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{7!}}$

$= 13 \cdot 11 \cdot 3 \cdot 4$

$= \boxed{1716}$

b.) 13 total = 7 woman + 6 men

i. $\binom{7}{4}\binom{6}{3} = \left(\dfrac{7!}{(7-4)!\,4!}\right)\left(\dfrac{6!}{(6-3)!\,3!}\right) = 700$

ii. total - All woman = 1716 - 1 = 1715

iii. at most 3 woman

1

2

3

$\binom{7}{1}\binom{6}{6} + \binom{7}{2}\binom{6}{5} + \binom{7}{3}\binom{6}{4} = \boxed{165}$

c.) 13

2 will not work together = A, B

$\binom{11}{6} \times 2 + \binom{11}{7} = 1254$

d.) 2 people either work together or won't

2 on committe

$$\binom{11}{5} + \binom{11}{7}$$

(9.7) Pascal's Formula & Binomial Theorem

$$\boxed{\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}}$$ where $n, r \in \mathbb{Z}^+$ & $r \leq n$

pascal's formula

*Proof for Pascal's Formula:

$$RHS = \binom{n}{r} + \binom{n}{r-1} = \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-(r-1))!\,(r-1)!} = \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!\,r + n!\,(n-r+1)}{(n-r+1)!\,r!} = \frac{n!\,(r + (n-r+1))}{(n-r+1)!\,r!}$$

$$= \frac{n!\,(n+1)}{(n-r+1)!\,r!} = \frac{(n+1)!}{(n+1-r)!\,r!} = LHS$$

Pg 603 7.) $\binom{n+3}{n+1} = \frac{(n+3)(n+2)}{2} \quad n \geq -1$

Proof:

$$LHS = \binom{n+3}{n+1} = \frac{(n+3)!}{(n+3-(n+1))!\,(n+1)!} = \frac{(n+3)(n+2)\cancel{(n+1)!}}{2!\,\cancel{(n+1)!}}$$

$$= \frac{(n+3)(n+2)}{2} = RHS$$

Why pascal's Formula makes sense.....

• recall pascal's triangle

0 1 2 3 4 ....

1

1 1

1 2 1

1+3=4 → 1 3 3 1

1 4 6 4 1

⋮

$\binom{4}{1} = \frac{4!}{3! \cdot 1!} = 4$

$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \binom{n}{3} \ldots$

* Binomial Theorem:

$(\overset{x}{a} + \overset{y}{b})^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-1} y^k = x^n + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \ldots$

$\binom{n}{n-1} x^1 y^{n-1} + y^n$

24.) $(u^2-3v)^4 = ((\overset{x}{u^2}) + (\overset{y}{-3v}))^4$

↳ $\binom{4}{0}(u^2)^4(-3v)^0 + \binom{4}{1}(u^2)^3(-3v)^1 + \binom{4}{2}(u^2)^2(-3v)^2$

$+ \binom{4}{3}(u^2)^1(-3v)^3 + \binom{4}{4}(u^2)^0(-3v)^4 = \boxed{u^8 - 12vu^6 + 54v^2u^4 - 108v^3u^2 + 81v^4}$

HW: Pg 581 1, 5, 6, 9, 10, 13, 16

603 1, 3, 6, 10, 12, 13, 19, 21, 23, 26, 29, 31, 33, 43