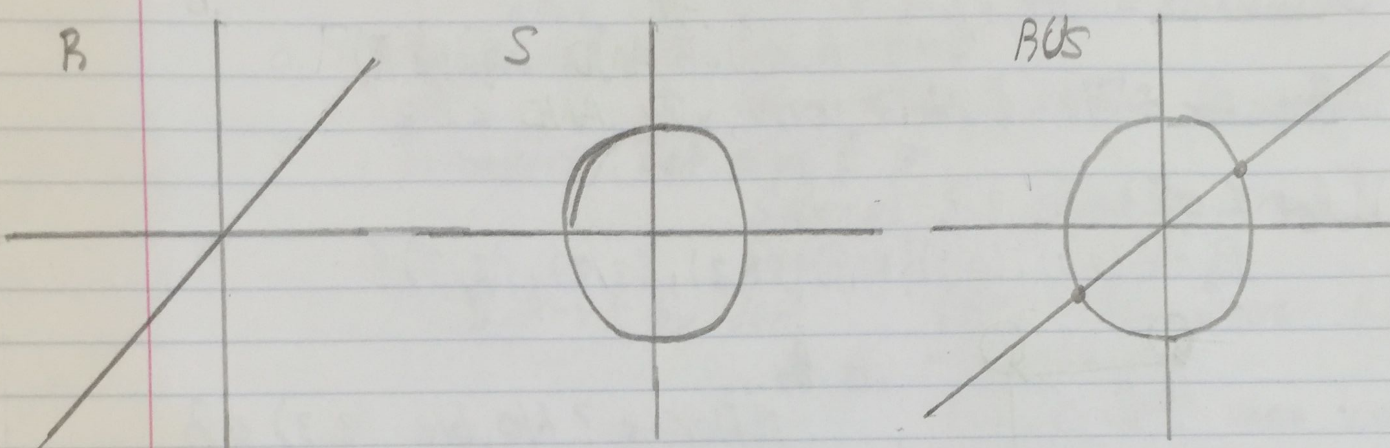
*Test on 21st (Thursday)

4/14/16

Pg 449 22.) $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 4\}$
$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y\}$ > graph



$R \cap S = \{(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})\}$

$x^2 + y^2 = 4$

$x = y$

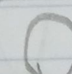
$2x^2 = 4 \Rightarrow x = \pm\sqrt{2}$

(8.2) $\boxed{\text{Reflexivity, Symmetry, \& Transitivity}}$

*Reflexive:

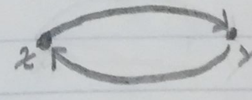
$\boxed{\text{A relation is reflexive} \iff \forall x \in A, (x, x) \in \mathbb{R}}$

graphs



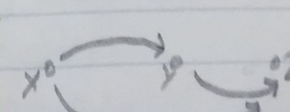
*Symmetry:

$\boxed{\begin{array}{l} \mathbb{R} \text{ is symmetric} \iff \forall x, y \in A \\ \text{If } (x, y) \in \mathbb{R} \text{ then } (y, x) \in \mathbb{R} \end{array}}$



*Transitive:

$\boxed{\begin{array}{l} \mathbb{R} \text{ is transitive} \iff \forall x, y, z \in A \\ \text{If } (x, y) \in \mathbb{R} \text{ And } (y, z) \in \mathbb{R}, \text{ then } (x, z) \in \mathbb{R} \end{array}}$



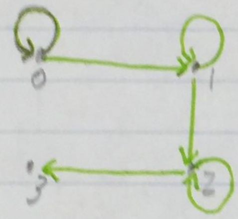
* Negations:

① Reflexive = If not reflexive $\exists x \in A \ni x \not{R} x$ $((x,x) \notin R)$

② Symmetric = $\exists x, y \in A \ni x R y$ AND $y \not{R} x$
$((x,y) \in R$ AND $(y,x) \notin R)$

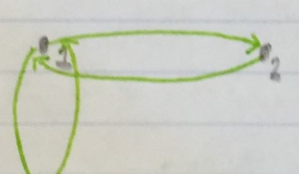
③ Transitive = $\exists x, y, z \in A \ni x R y, y R z,$ AND $x \not{R} z$

Pg 458 2.) let $A = \{0, 1, 2, 3\}$
$R_2 = \{(0,0), (0,1), (1,1), (1,2), (2,2), (2,3)\}$



is R_2...
reflexive? No, b/c $(3,3) \notin R$
Symmetric? No, $(0,1) \in R_2$ BUT $(1,0) \notin R_2$
Transitive? No, $0R1$ and $1R2$ but $0 \not{R} 2$

4.) $R_4 = \{(1,2), (2,1), (1,3), (3,1)\}$



is R_4...
reflexive? No, there are no loops
Symmetric? Yes, $1R2$ & $2R1$, $1R3$ and $3R1$
Transitive? No, $3R1$ & $1R2$ BUT $3 \not{R} 2$

* Infinite Sets:

$\boxed{\begin{array}{l} \forall x, y \in \mathbb{R}, \text{ if } xRy \text{ then } yRx \\ \forall x, y \in \mathbb{R}, \text{ if } x = y \text{ then } y = x \end{array}}$

10.) C = the circle relation on $\mathbb{R}$; $\forall x, y \in \mathbb{R}$ $x C y \Leftrightarrow x^2 + y^2 = 1$

Reflexive? No b/c $1 \not{C} 1$
Symmetric? Yes, Symmetric (if $x^2+y^2=1$ then $y^2+x^2=1$)
Transitive? No, $(1/2, \sqrt{3}/2) \in C$ & $(\sqrt{3}/2, 1/2) \in C$ BUT $(1/2, 1/2) \notin C$

13.) F on $\mathbb{Z}$
$\forall m,n \in \mathbb{Z}$ $\quad mFn \iff 5|m-n$

Reflexive? yes, let $m=n=0$
$(m-n)=5r=0 \rightarrow 5|0$ so yes

Symmetric? Let $\cancel{x},y \in \mathbb{Z} \ni$
$5|(x-y)$ [need to show]
$5|(y-x)$

$5|(x-y)$ by def $\quad (x-y)=5r$ where $r \in \mathbb{Z}$
$-(y-x)=5r$
$(y-x)=5(-r)$ where $-r \in \mathbb{Z}$

So by def $5|(y-x) \Rightarrow$ Yes!

Transitive? let $z,y,z \in \mathbb{Z}$
$5|(x-y)$ and $5|(y-z)$
$5|(x-z)$ [need to show]

$x-y=5r$ & $y-z=5s$ where $r,s \in \mathbb{Z}$

$$\begin{array}{r} x-y=5r \\ \underline{y-z=5s} \\ x-z=5r+5s \end{array}$$

$x-z=5(r+s)$ where $r+s \in \mathbb{Z}$

$\therefore 5|(x-z)$ so yes!

14.) O defined on $\mathbb{Z}$
$\forall m,n \in \mathbb{Z}$, $\quad mOn \iff m-n$ is odd

Reflexive? No, let $m=n=0$
$m-n=0$ which is even so n.

Symmetric? Yes, $\quad m-n=2r+1$ by def, show $n-m=2s+1$
$-(n-m)=2r+1$
$n-m=-2r-1+1-1=-2r-2+1=2(-r-1)+1$, which is odd.

Transitive? NO, $m-n = 2r+1$ & $n-p = 2s+1$ need to show $m-p = 2t+1$

$$m-n = 2r+1$$
$$\underline{n-p = 2s+1}$$
$$m-p = 2r+2s+2$$
$$m-p = 2(r+s+1)$$ which is even, so no!

Equivalence Relations

* The Relation Induced by the partition, B on A =

> $\forall x, y \in A \ldots.$
> $x B y \iff$ there is a subset A_i of the partition $\ni x, y \in A_i$

- **THEOREM** Let A be a set with a partition AND let B be the relation induced by the partition. If this is true, then B is reflexive, symmetric, & transitive. We call this B an equivalence relation.

475 2.) The partitions $\{0,1,2,3,4\}$ induces a relation B on $\{0,1,2,3,4\}$. Find all the ordered pairs in B.

b. $\{0\}, \{1,3,4\}, \{2\}$

$B = \{(0,0),(1,1),(3,3),(4,4),(1,3),(3,1),(1,4),(4,1),(3,4),(4,3),(2,2)\}$

* Equivalence Classes:

> Suppose A is a set and B is an equivalence relation on A. For each element $a \in A$, the equivalence class of a ($[a]$) and called the class of a, which is the set of all elements $x \in A \ni x$ is related to a by B.
>
> $$\boxed{[a] = \{x \in A \mid x B a\}}$$

- Lemmas
  1. If $aBb \Rightarrow [a] = [b]$
  2. Either $[a]=[b]$ or $[a] \cap [b] = \emptyset$

4.) $A = \{a, b, c, d\}$ $R = \{(a,a), (b,b), (b,d), (c,c), (d,b), (d,d)\}$

$[a] = \{a\}$

$[b] = \{b, d\}$ since $d \in [b] \Rightarrow [d] = [b]$

$[c] = \{c\}$