4/12/16

(7.2) One-to-One & Onto

*One-to-One: Let F be a Function from set X to set Y

F is "one-to-one" (injective)

iff

$\forall x_1, x_2 \in X$ if $F(x_1) = F(x_2)$ then $x_1 = x_2$

if $x_1 \neq x_2$ then $F(x_1) \neq F(x_2)$

other way of defining → • $F: X \to Y$ is one-to-one $\iff \forall x_1, x_2 \in X$

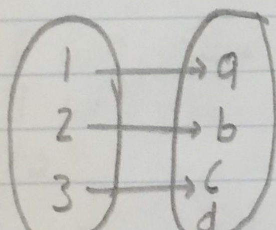
If $F(x_1) = f(x_2)$ then $x_1 = x_2$

• THE NEGATION:

$F: X \to Y$ is NOT one-to-one if $\exists\, x_1, x_2 \in X \ni$

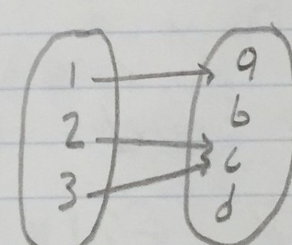
$F(x_1) = F(x_2)$ and $x_1 \neq x_2$

ex.)



1-1 ✓

,



NOT 1-1

*Onto: $F: X \to Y$ is onto (surjective) $\iff \forall y \in Y\ \exists x \in X$

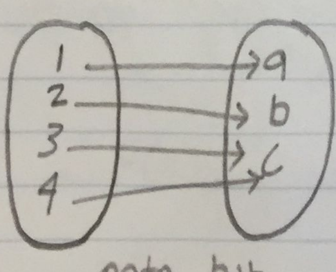
$\ni F(x) = y$

• THE NEGATION:

$F: X \to Y$ is NOT onto if $\exists y \in Y \ni \forall x \in X$

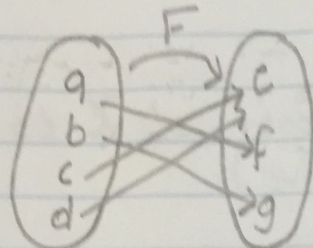
$F(x) \neq y$

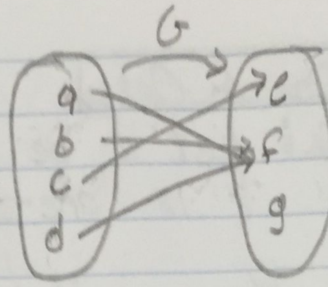
ex.)



onto but
NOT 1-1

Pg 414 7.)





<u>one to one?</u> NO, both c and d map to e

No

<u>onto?</u> Yes, each element in Co-domain is an image of an element in the domain

NO

<u>Proving One-to-one & Onto</u>

* To prove a function to one-to-one a direct proof can be used.
  - Suppose $x_1, x_2 \in X \ni f(x_1) = f(x_2)$, then <u>SHOW</u> $x_1 = x_2$

Pg 414 12a.) $F: \mathbb{Z} \to \mathbb{Z}$ $\quad F(n) = 2 - 3n \ \forall n \in \mathbb{Z}$

<u>Proof</u>: $F(x_1) = 2 - 3x_1 \qquad F(x_2) = 2 - 3x_2$

$$2 - 3x_1 = 2 - 3x_2$$

$$x_1 = x_2$$

$\therefore$ This proves $F(n)$ is 1-1

disproof → ex.) $G: \mathbb{Z} \to \mathbb{Z}$ $\quad G(n) = n^2 \ \forall n \in \mathbb{Z}$

"This isn't 1-1 so to prove this find two different elements in the domain that map to the same element in the codomain."

<u>Proof</u>: $G(2) = 4$, $G(-2) = 4$

$\therefore$ $G(n)$ is NOT one-to-one

* To prove onto use generalizing from the generic particular
- Suppose $y \in Y$ then SHOW $\exists x \in X$ with $F(x) = y$

* To disprove onto, find an element $y \in Y \ni y \neq F(x)$ for any $x \in X$.

Ex.) $F: \mathbb{Z} \to \mathbb{Z}$ $F(n) = 2 - 3n$ $\forall n \in \mathbb{Z}$

Proof: Suppose $F(n) = 0$

$$0 = 2 - 3n$$
$$3n = 2$$
$$n = 2/3$$

Which is a contradiction, since $2/3$ is not an integer. $\therefore$ $F(n)$ is not onto.

What if we change F so that $F: \mathbb{R} \to \mathbb{R}$

- Still will be one-to-one
- onto?

Proof: It is onto,

$$y = 2 - 3x$$
$$3x = 2 - y$$
$$x = \frac{2 - y}{3}$$

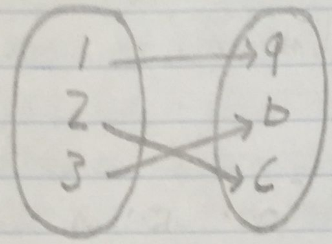
$$F(x) = F\left(\frac{2-y}{3}\right)$$
$$= 2 - 3\left(\frac{2-y}{3}\right)$$
$$= 2 - 2 + y$$
$$= y$$

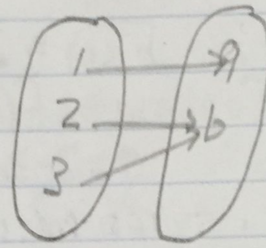
$\therefore F(x) = y$

$\boxed{\text{Bijections}}$ (one-to-one correspondence)

* DEF: A one-to-one correspondence (or bijection) from X to Y is a function $F: X \to Y$ that is both one-to-one and onto.

ex.)

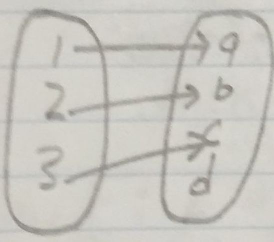




*Inverse IS a function!)

one-to-one
onto
=
Bijection

not one-to-one
onto
* Inverse not a function because of b



one-to-one
not onto
* Inverse is NOT a function because of d

* THEOREM: Suppose $F: X \to Y$ is a one-to-one correspondence then $\exists F^{-1}: Y \to X \ni$ for any $y \ni Y$.

- $F^{-1}(y) =$ a unique element $x \in X \ni F(x) = y$
- F^{-1} is called the inverse function for F.

Pg 414 18.) $F(x) = \frac{x+1}{x-1}$ $\forall x \in \mathbb{R}$ where $x \neq 1$

Proof: $F(x_1) = \frac{x_1+1}{x_1-1}$ $\quad F(x_2) = \frac{x_2+1}{x_2-1}$

$$\frac{x_1+1}{x_1-1} = \frac{x_2+1}{x_2-1}$$

HW: Pg 413 #'s 1-6, 8-11, 15, 16, 23, 42, 44-47

$$x_1x_2 - x_1 + x_2 - 1 = x_2x_1 - x_2 + x_1 - 1$$
$$-x_1 + x_2 = -x_2 + x_1$$
$$2x_2 = 2x_1$$
$$x_2 = x_1 \checkmark$$

Chapter 8: Relations

(8.1) $\boxed{\text{Relations on Sets}}$

* Binary Relations = Cartesian products of 2 sets (b/c of 2 sets).
  - For n sets they are called n-ary relations (whenever the book uses relations they mean binary relations)

* Notation:

Notation of a Relation $\mathcal{R}$ from set X to set Y is $\boxed{x \mathcal{R} y}$ where $x \in X$ and $y \in Y$.

Pg 448 1.) $m E n \iff m-n$ is even where E is from $\mathbb{Z}$ to $\mathbb{Z}$

a. Is $0E0$? yes b/c $0-0=0$, and 0 is even

b. Is $5E2$? No b/c $5-2=3$ and 3 is not even.

c. Is $\underbrace{(6,6)\in E}_{\text{means } 6E6}$? yes

d. Is $(-1,7)\in E$? yes b/c $-1-7=-8$ which is even

recall, power set is the set containing all subsets

6.) $X=\{a,b,c\}$ define J on $P(x)$; $\forall A,B\in P(x)$ $AJB \iff A\cap B\neq\emptyset$

a. $\{a\} \not J \{b\}$ X

b. $\{a,b\} J \{b,c\}$ ✓

c. $\{a,b\} J \{a,b,c\}$ ✓

* Inverse of a Relation:

Let $\mathcal{R}$ be a relation from A to B, the inverse $\mathcal{R}^{-1}$ from B to A will be...

$$\mathcal{R}^{-1}=\{(y,x)\in B\times A \mid (x,y)\in\mathcal{R}\}$$

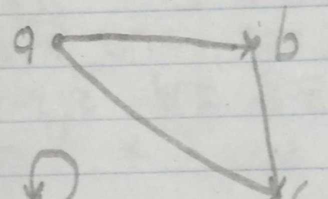
11.) $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$ $\forall (x, y) \in A \times B$
$x S y \Leftrightarrow x | y$

$S = \{(3,6), (4,4), (5,5)\}$

$S^{-1} = \{(6,3), (4,4), (5,5)\}$

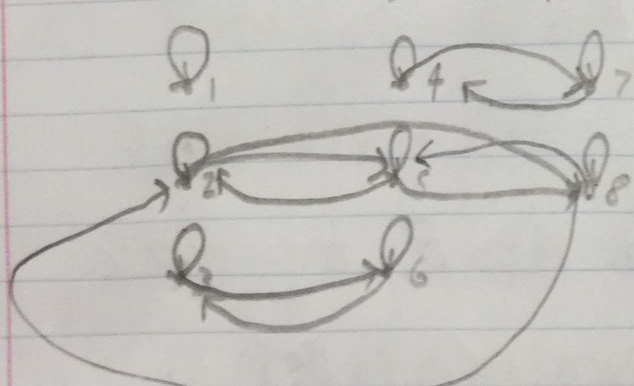
* Directed Graphs (Digraphs)
Shows the relation R from X to Y using an arrow that leads from some $x \in X$ to some $y \in Y$ iff $(x, y) \in R$

9 14.) $B = \{a, b, c, d\}$, $S = \{(a,b), (a,c), (b,c), (d,d)\}$



17.) $A = \{2, 3, 4, 5, 6, 7, 8\}$ $x T y \Leftrightarrow 3 | (x - y)$

$T = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7)$
$(5,2), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7)$
$(8,2), (8,5), (8,8)\}$



HW: Pg 448 #'s 3, 4, 5, 7, 8, 9, 10, 12, 13, 15, 16, 19, 21, 23