4/7/16

Proofs Using Set Identities (Algebraic Proofs)

Pg 373 32.) $(A-B)\cup(A\cap B)=A$

$$\begin{aligned}(A-B)\cup(A\cap B) &= (A\cap B^c)\cup(A\cap B)\\ &= A\cap(B^c\cup B)\\ &= A\cap U\\ &= A\end{aligned}$$

35.) $A-(A-B)=A\cap B$

$$\begin{aligned}A-(A-B) &= A-(A\cap B^c)\\ &= A\cap(A\cap B^c)^c\\ &= A\cap(A^c\cup B)\\ &= (A\cap A^c)\cup(A\cap B)\\ &= \emptyset\cup(A\cap B)\\ &= A\cap B\end{aligned}$$

HW: Pg 372 #'s 1, 3, 6, 12, 13, 30, 31, 34, 37

36.) $((A^c\cup B^c)-A)^c=A$

$$\begin{aligned}((A^c\cup B^c)-A)^c &= ((A^c\cup B^c)\cap A^c)^c\\ &= ((A\cap B)^c\cap A^c)^c\\ &= ((A\cap B)^c)^c\cup(A^c)^c\\ &= (A\cap B)\cup A\\ &= A\end{aligned}$$

Chapter 7: Functions

.1) **Functions Defined on General Sets**

* Relation = A rule that relates one set to another.
* Function ($f: X \to Y$) = A relation from X, the domain, to Y, the co-domain ∋...
  - Every element in X is related to some element in Y

    AND

  - No element in X is related to more than 1 element in Y.
* "F(x)" is the output for an input of x.
  - Same thing as... The value of f at x

    The image of x under f.
* "F" is a function.
* The Range = The set of all values of f.

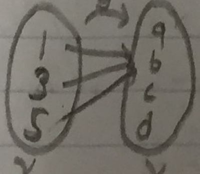
$$\boxed{\{y \in Y \mid f(x) = y \text{ for some } x \in X\}}$$

* The Inverse = If $f(x) = y$ then x is called a preimage of y or inverse image of y.

$$\boxed{y = \{x \in X \mid f(x) = y\}}$$

how to map the function ↓

393 2.) let $X = \{1, 3, 5\}$ where $Y = \{a, b, c, d\}$, define $g: X \to Y$



a. $\{1, 3, 5\}$ = Domain

$\{a, b, c, d\}$ = Co-domain

b. $g(1) = b$

$g(3) = b$

$g(5) = b$

c. range $= \{b\}$

d. Is 3 an inverse image of a? no
Is 1 an inverse image of b? Yes
e. What is the inverse image of b?
$\{1, 3, 5\}$
What is the inverse image of c?
$\emptyset$
f. Represent g as a set of ordered pairs.

$$\{(1, b), (2, b), (5, b)\}$$

Types of Functions

① Identity Function:

$$I_X(x) = x \quad \forall x \in X$$

Pg 394 5c.) $I_Z(K(t)) = K(t)$

② Sequences:

6b.) $0, -2, 4, -6, 8, -10$

$$F: Z^{nonneg} \to Z \quad \boxed{n = (-1)^n 2n}$$

③ Functions Defined on a Power Set:

Pg 394 7b.) $A = \{1, 2, 3, 4, 5\}$, define $F: P(A) \to Z$

$\forall_{sets}$ x in $P(a)$....

$$F(x) = \begin{cases} 0 & \text{if } x \text{ has an even number of elements} \\ 1 & \text{if } x \text{ has an odd number of elements} \end{cases}$$

b.) $F(\emptyset) = 0$
c.) $F(\{2,3,4,5\}) = 0$

④ Functions Defined on a Cartesian Product:

11.) Define $F: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ for all ordered pairs (a,b) of integers $F(a,b) = (2a+1, 3b-2)$

c. $F(3,2) = (2(3)+1, 3(2)-2)$
$= (7,4)$

12.) $G: J_5 \times J_5 \to J_5 \times J_5 \quad \forall (a,b) \in J_5 \times J_5$

$G(a,b) = ((2a+1) \bmod 5, (3b-2) \bmod 5)$

c. $G(3,2) = ((2(3)+1) \bmod 5, (3(2)-2) \bmod 5)$
$= (7 \bmod 5, 4 \bmod 5)$
$= (2, 4)$

Functions that are not well defined:
* These fail to satisfy atleast 1 requirement to be a function.

Pg 395 | 34.) C says $h: \mathbb{Q} \to \mathbb{Q}$ $h(m/n) = m^2/n$ $\forall m,n \in \mathbb{Z}$ $n \neq 0$.
D says this isn't well defined. Why is D right?
- B/c you have 1 element in the domain mapped to 2 different elements in the co domain.

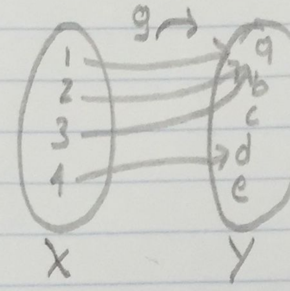
39.) $X = \{1, 2, 3, 4\}$

$Y = \{a, b, c, d, e\}$

define $g: X \to Y$

$g(1) = a$
$g(2) = a$
$g(3) = a$
$g(4) = d$

a.



DEF

If $F: X \to Y$ is a function & $A \subseteq X$ & $C \subseteq Y$
then $F(A) = \{y \in Y \mid y = F(x)$ for some $x \in A\}$
and

F inverse ⟶ $F^{-1}(C) = \{x \in X \mid F(x) \in C\}$

b. $A = \{2, 3\}$ $C = \{a\}$ $D = \{b, c\}$

$g(A) = \{a\}$
$g(X) = \{a, d\}$
$g^{-1}(C) = \{1, 2, 3\}$
$g^{-1}(D) = \emptyset$
$g^{-1}(Y) = X$
$= \{1, 2, 3, 4\}$

HW: Pg 393 1, 3, 5, 6, 7ac, 8, 9, 11ab, 13, 17, 25