4/5/16

Power Sets

DEF → The power set ($\mathcal{P}$) is the set of all subsets in a given set.

Pg 351 31.) $A=\{1,2\}$, $B=\{2,3\}$

a. $P(A\cap B) = \{\phi, \{2\}\}$

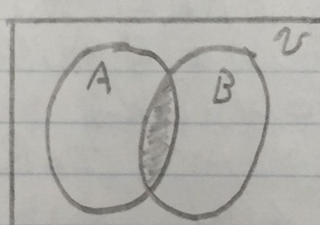
$\hookrightarrow A\cap B=\{2\}$

b. $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

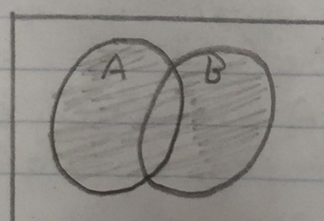
c. $P(A\cup B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

(6.2) **Properties of Sets**

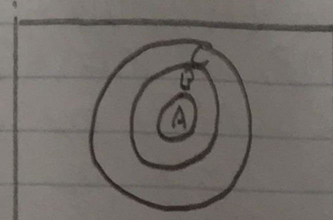
* Theorem ① Inclusion of Intersection: $A\cap B \subseteq A$ and $A\cap B\subseteq B$



② Inclusion of Union: $A\subseteq A\cup B$ and $B\subseteq A\cup B$



③ Transitive Property of Subsets: If $A\subseteq B$ and $B\subseteq C$ then $A\subseteq C$



Set Definitions for Proofs

① $x \in A \cup B \iff x \in A$ or $x \in B$
② $x \in A \cap B \iff x \in A$ and $x \in B$
③ $x \in A - B \iff x \in A$ and $x \notin B$
④ $x \in A^c \iff x \notin A$
⑤ $(a,b) \in A \times B \iff a \in A$ and $b \in B$
⑥ $A \subseteq B \iff$ if $x \in A$ then $x \in B$

Proving Sets are Equal To Eachother

* To do this, we must show that both sets are subsets of eachother.
* So to Show $A=B$ you need to prove BOTH $A \subseteq B$ and $B \subseteq A$

265 6.) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Part 1: (Show $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$)
Suppose $x \in A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$ by definition. Also, $x \in B$ or $x \in C$ by definition

Case 1 (we know $x \in A$)
Suppose $x \in B$.
Since $x \in A$ and $x \in B$, by definition $x \in A \cap B$

Case 2 (we know $x \in A$)
Suppose $x \in C$.
Since $x \in A$ and $x \in C$, by definition $x \in A \cap C$

So we know by case 1, $x \in A \cap B$ or by case 2, $x \in A \cup C$.
$\therefore$ By definition, $x \in (A \cap B) \cup (A \cap C)$

Part 2: (Show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$)
Suppose $x \in (A \cap B) \cup (A \cap C)$ by definition.
Then $x \in (A \cap B)$ or $x \in (A \cap C)$ " "

Case 1 Suppose $x \in (A \cap B)$ by definition.
$x \in A$ and $x \in B$ by definition.
Since $x \in B$ then $x \in (B \cup C)$ by definition
$\therefore$ $x \in A$ and $x \in (B \cup C)$ So by definition,
$x \in A \cup (B \cup C)$ "

When a ~~[illegible]~~ proving in the future, write WHAT each part is proving first

$\boxed{\text{Case 2}}$ Suppose $x \in A \cup C$. By definition, $x \in A$ and $x \in C$.
Since $x \in C$ then $x \in (B \cup C)$
$\therefore$ $x \in A$ and $x \in (B \cup C)$ so by definition,
$x \in A \cap (B \cup C)$

10.) Prove $(A-B) \cap (C-B) = (A \cap C) - B$

1. (Left ⊆ Right) part 1: Suppose $x \in (A-B) \cap (C-B)$. Then by definition,
$x \in (A-B)$ and $x \in (C-B)$.
So $x \in A$ and $x \notin B$ by definition.
Also, $x \in C$ and $x \notin B$ by definition.
$\therefore$ It can be said $x \in (A \cap C) - B$

(right ⊆ left) part 2: Suppose $x \in (A \cap C) - B$ by def. Then by def
$x \in (A \cap C)$ and $x \notin B$.
$x \in A$ and $x \in C$ and $x \notin B$.
Since $x \in A$ and $x \notin B$, then $x \in (A-B)$
AND since $x \in C$ and $x \notin B$, then $x \in (C-B)$
$\therefore$ $x \in (A-B) \cap (C-B)$

15.) Prove $\forall$ sets A and B, IF $A \subseteq B$ then $B^c \subseteq A^c$

Proof: By def, when $x \in A$, $x \in B$.
Suppose $x \in B^c$ then by def $x \notin B$.
So since $x \notin B$ then $x \notin A$.
So $x \in A^c$. $\therefore$ $B^c \subseteq A^c$

HW: Pg 364 #'s 1,2,4,9,13,23

Next Test will most likely go up to 8.3

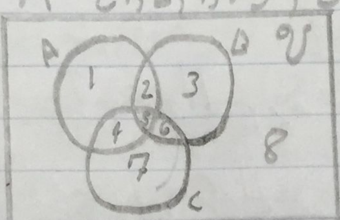
using venn diagrams makes this easy!

(6.3) $\boxed{\text{Disproofs \& Algebraic Proofs}}$

* Disproofs work the same way they always have.

Pg 372 7.) $\forall$ sets A, B, C, $(A-B)\cap(C-B) = A-(B\cup C)$

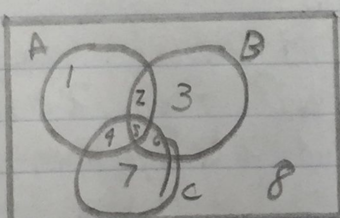
Set $A=\{1,2,4,5\}$, $B=\{2,3,5,6\}$, $C=\{4,5,6,7\}$ & $U=\{1\to 8\}$



$(A-B)\cap(C-B) = \{1,4\}\cap\{4,7\}$
$\{4\}$
$A-(B\cup C) = \{1,2,4,5\}-\{2,3,4,5,6,7\}$
$= \{1\}$
$\therefore$ This is false

5.) $A-(B-C) = (A-B)-C$

" "



$A-(B-C) = \{1,2,4,5\}-\{3,2\}$
$= \{1,4,5\}$
$(A-B)-C = \{1,4\}-\{7,4,5,6\}$
$= \{1\}$
False!

$\boxed{\text{Set Identities}}$ (DONT NEED TO MEMORIZE NAMES)

1. $A\cup B = B\cup A$, $A\cap B = B\cap A$
2. $(A\cup B)\cup C = A\cup(B\cup C)$ → same for $\cap$
3. $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$
4. $A\cup\emptyset = A$, $A\cap\emptyset = \emptyset$
5. $A\cup A^c = U$, $A\cap A^c = \emptyset$
6. $(A^c)^c = A$
7. $A\cup U = U$
8. $(A\cup B)^c = A^c\cap B^c$, $(A\cap B)^c = A^c\cup B^c$
9. $U^c = \emptyset$, $\emptyset^c = U$
10. $A-B = A\cap B^c$
11. $A\cup(A\cap B) = A$, $A\cap(A\cup B) = A$

HW: Pg 372 1, 6, 12, 13, 30, 34, 37