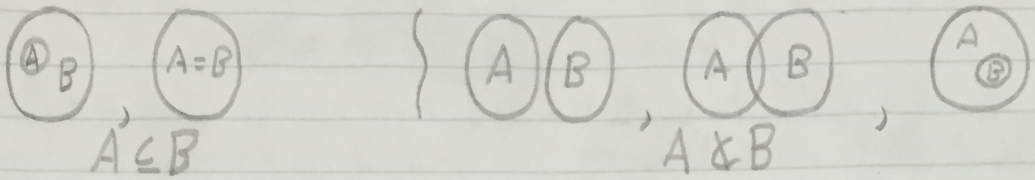
3/31/16

Venn Diagrams

* Def = Relationships between A & B denoted by pictures.

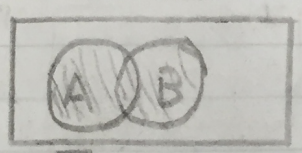


Operations on Sets

* Universal Set ($\mathcal{U}$) = The set of all elements considered
* Let "A" & "B" be subsets of the Universal set...

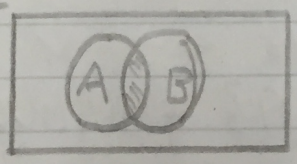
① The Union: $A \cup B = \{x \in \mathcal{U} \mid x \in A \text{ or } x \in B\}$

↘ Union "Combination of sets"



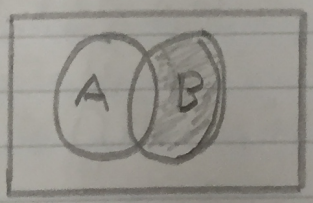
② The Intersection: $A \cap B = \{x \in \mathcal{U} \mid x \in A \text{ and } x \in B\}$

↘ Intersection "What both sets have in common"

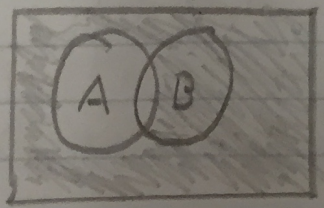


③ The Difference: $A - B = \{x \in \mathcal{U} \mid x \in B \text{ and } x \notin A\}$

↘ minus "What is only in A"



④ The Complement: $A^c = \{x \in \mathcal{U} \mid x \notin A\}$



* Disjoint Sets = Occurs when A & B have nothing in common ($A \cap B = \emptyset$)

Pg 350 10.) $A = \{1, 3, 5, 7, 9\}$
$B = \{3, 6, 9\}$
$C = \{2, 4, 6, 8\}$

a. $A \cup B = \{1, 3, 5, 6, 7, 9\}$
b. $A \cap B = \{3, 9\}$
c. $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
d. $A \cap C = \emptyset$
e. $A - B = \{1, 5, 7\}$
f. $B - A = \{6\}$
g. $B \cup C = \{2, 3, 4, 6, 8, 9\}$
h. $B \cap C = \{6\}$

Ex.) $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{1, 3, 5, 7, 9\}$
$B = \{3, 6, 9\}$
$C = \{2, 4, 6, 8\}$

① $A^c = U - A = \{0, 2, 4, 6, 8, 10\}$
② Find $B \cap C = \{6\}$
③ $A \cup (B \cup C) = \{1, 3, 5, 6, 7, 9\}$
④ $A \cup B = \{1, 3, 5, 6, 7, 9\}$
⑤ $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
⑥ $(A \cup B) \cap (A \cup C) = \{1, 3, 5, 6, 7, 9\} = A \cup (B \cup C)$
⑦ $A^c \cup B^c = \{0, 2, 4, 8, 10\}$
⑧ $(A \cup B)^c = \{1, 3, 5, 6, 7, 9\}^c = \{0, 2, 4, 8, 10\}$

There's a distributive Property!

Demorgan's Law!

12.) $A = \{x \in \mathbb{R} \mid -3 \leq x \leq 0\}$
$B = \{x \in \mathbb{R} \mid -1 < x < 2\}$
$C = \{x \in \mathbb{R} \mid 6 < x \leq 8\}$

a. $A \cup B = \{x \in \mathbb{R} \mid -3 \leq x < 2\} = [-3, 2)$
b. $A \cap B = \{x \in \mathbb{R} \mid -1 < x \leq 0\} = (-1, 0]$
c. $A^c = (-\infty, -3) \cup (0, \infty) = \{x \in \mathbb{R} \mid x < -3 \text{ or } x > 0\}$
f. $B^c = (-\infty, -1] \cup [2, \infty)$
g. $A^c \cap B^c = \{x \in \mathbb{R} \mid x < -3 \text{ or } x > 2\} = (-\infty, -3) \cup (2, \infty)$
h. $A^c \cup B^c = (-\infty, -1] \cup (0, \infty)$
i. $(A \cap B)^c = A^c \cup B^c$
j. $(A \cup B)^c = A^c \cap B^c$

} Demorgan

Notations for Multiple Sets

* Given $A_0, A_1, A_2 \ldots \ni A_i \subseteq U \quad i \in Z^{\text{nonnegative}}$

$\bigcup_{i=0}^{n} A_i = \{x \in U \mid x \in A_i \text{ for atleast one } i = 0, 1, 2 \ldots\}$

$\bigcup_{i=0}^{\infty} A_i = \{x \in U \mid x \in A_i \text{ for atleast one nonnegative integer}\}$

$\bigcap_{i=0}^{n} A_i = \{x \in U \mid x \in A_i \ \forall\ i = 0, 1, 2, 3 \ldots\}$

$\bigcap_{i=0}^{\infty} A_i = \{x \in U \mid x \in A_i \ \forall \text{ nonnegative integers } i\}$

*Mutually Disjoint = Means you have a collection of sets that have no elements common to any two of them. (non overlapping).

Partitions of Sets

* DEF = Finite or infinite collection of nonempty sets $\ni$
  1. $\bigcup A_i = A$
  2. $A_1, A_2, A_3 \ldots$ mutually disjoint.

Pg 350 20.) $B_i = \{x \in \mathbb{R} \mid 0 \le x \le i\} \ \forall i \in Z \ni i = 1, 2, 3, 4$

a. $B_1 \cup B_2 \cup B_3 \cup B_4 = \bigcup_{i=1}^{4} B_i = \{x \in \mathbb{R} \mid 0 \le x \le 4\}$

b. $\bigcap_{i=1}^{4} B_i = \{x \in \mathbb{R} \mid 0 \le x \le 1\}$

c. no they are not

23.) $V_i = \{x \in \mathbb{R} \mid -1/i \le x \le 1/i\} = [-1/i, 1/i] \quad i \in Z^+$

a. $\bigcup_{i=1}^{4} V_i = \{x \in \mathbb{R} \mid -1 \le x \le 1\}$

d. $\bigcup_{i=1}^{\infty} V_i = [-1, 1]$

b. $\bigcap_{i=1}^{4} V_i = \{x \in \mathbb{R} \mid -1/4 \le x \le 1/4\}$

g. $\bigcap_{i=1}^{\infty} V_i = \{0\}$

c. No!

HW: Pg 349 9, 11, 13, 15, 17, 19, 21, 22, 2
28, 31, 34