3/29/16

Chapter 1: Speaking Mathematically

(1.2) The Language of Sets

*Let "S" be a set
(in general we use capital letters to represent the names of sets.)

$x \in S$, x is an element of set S

$x \notin S$, x is NOT an element of S.

*Roster Method:
- List of elements in a set.

ex.) $A = \{1, 2, 3, 5, 7, 9, 12\}$

These are elements of the set

- Order in a set doesn't matter

ex.) $\{1, 3, 2\} = \{3, 1, 2\}$

- In general, A=B iff A & B contain the EXACT same elements.

*Set Builder Notation:

$\{x \in S \mid P(x)\}$ condition in which x will be contained in the set

"such that"

ex.) $\{x \in \mathbb{Z}^{+} \mid x \text{ is even}\} = \{0, 2, 4, 6, \ldots\}$

Set Builder Roster

ex.) $\{x \in \mathbb{R} \mid x \geq 4\}$ (can't use Roster for this!)

* Subsets:

Notation → $A \subseteq B \Rightarrow$ "A is a subset of B"

- $A \subseteq B$ iff every element of A is also an element of B.

ex.) $A = \{1, 2\}$ $B = \{1, 2\} \Rightarrow A \subseteq B \land B \subseteq A \Rightarrow A = B$

* Proper Subsets:

- These are subsets but not equal to the original set.

Notation → $A \subset B \Rightarrow$ "Each element of A is also in B b
$A \neq B$"

ex.) $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\} \Longrightarrow A \subseteq B$ also $A \subset B$ but $B \nsubseteq A$

- Every set is a Subset of itself!

* The Empty Set (The "Null" Set):

- This is the set that contains no elements.

$\{\emptyset\}$ is not an set! "$\emptyset$" is really a symbol!!!

Notation → $\{\ \}$ OR $\emptyset$

- The empty set is a subset of every set.

of Elements in the set	set	Subsets	# of sets
0	$\emptyset$	$\emptyset$	1
1	$\{a\}$	$\emptyset, \{a\}$	2
2	$\{a, b\}$	$\emptyset, \{a\}, \{b\}, \{a, b\}$	4
3	$\{a, b, c\}$	$\emptyset, \{a\}, \{b\}, \{c\}, \{a, b, c\}$ $\{a, b\}, \{a, c\}, \{b, c$	8
4			16

2^n!

- A set containing n elements will have $\boxed{2^n}$ subsets and $\boxed{2^n - 1}$ proper subsets.

Cartesian Product (Cross Product):

$\boxed{\text{Notation}} \rightarrow A \times B \Rightarrow$ "A cross B" } ORDER MATTERS!

- This the set of all ordered pairs (a,b) where $a \in A$ and $b \in B$
- Using Set Builder Notation....

$\boxed{\text{Notation}} \rightarrow A \times B = \{a, b \mid a \in A \text{ and } b \in B\}$

3 12.) $S = \{2, 4, 6\}$ $T = \{1, 3, 5\}$

a.) $S \times T = \{(2,1), (2,3), (2,5), (4,1), (4,3), (4,5), (6,1), (6,3), (6,5)\}$
b.) $T \times S = \{(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)\}$
c.) $S \times S = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$

HW: Pg 13 #'s 1, 3, 5, 7, 9, 10, 11

Chapter 6: Set Theory

1) $\boxed{\text{Definition of the Element Method of Proof}}$

* Recall: $A \subseteq B \Leftrightarrow \forall x$ if $x \in A$, then $x \in B$

$A \not\subseteq B \Leftrightarrow \exists x \ni x \in A$ & $x \notin B$

* Element Argument:

- Use this to prove that a set is a subset of another set.
- Let A & B be given sets. To prove $A \subseteq B$...
  1. Suppose x is a particular (but arbitrary) element of A
  2. Show $x \in B$

350 4.) $A = \{n \in \mathbb{Z} \mid n = 5r, r = \mathbb{Z}\}$; $B = \{m \in \mathbb{Z} \mid m = 20s, s \in \mathbb{Z}\}$

a.) Is $A \subseteq B$? No!
$5 \in A$ But $5 \notin B$

b.) Is $B \subseteq A$? YES!

Proof: Suppose x is a particular but arbitrary element of B.
So $x = 20s$ (Algebraically manipulate)
$x = 5(4s)$
By closure of integers, $4s$ is an integer r
$X = 5r$
$\therefore X \in A$ which means $B \subseteq A$.

7.) $A = \{x \in \mathbb{Z} \mid x = 6a+4\}$; $B = \{y \in \mathbb{Z} \mid y = 18b-2$; $C = \{z \in \mathbb{Z} \mid z = 18c+16\}$
where $A, B, C \in \mathbb{Z}$.

a.) is $A \subseteq B$? NO!
Let $a = 1$, $X = 6(1)+4$
$= 10$
$10 = 18b - 2 \Rightarrow b = \frac{12}{18} \notin \mathbb{Z}$. So no

b.) is $B \subseteq A$? Yes!

Proof: Let x equal a particular but arbitrary element of B.
So $X = 18b - 2$
$X = 18b - 2 + 4 - 4$
$= 18b - 2 - 4 + 4$
$= 18b - 6 + 4$
$= 6(3b - 1) + 4$
By closure of integers, $3b - 2$ is some integer a
$= 6a + 4$
$\therefore B \subseteq A$ since $X \in A$

C.) Is $B=C$?

$\boxed{\text{Case 1}}$ $B \subseteq C$:

Let x be some particular but arbitrary element of B.

$$
\begin{aligned}
x &= 18b-2 \\
&= 18b-2+16-16 \\
&= 18b-16-2+16 \\
&= 18b-18+16 \\
&= 18(b-1)+16
\end{aligned}
$$

By closure of integers, $b-1$ is also an integer so

let $b-1=c$

$x = 18c+16$

$\therefore B \subseteq C$

$\boxed{\text{Case 2}}$ $C \subseteq B$:

" " C.

$$
\begin{aligned}
x &= 18c+16 \\
&= 18c+16+2-2 \\
&= 18(c+1)-2
\end{aligned}
$$

" ", $c+1$ " " so

let $c+1=b$

$x = 18b-2$

$\therefore C \subseteq B$