

- Use the Chain Rule to find  $\frac{dQ}{dt}$ , where  $Q = \sqrt{3x^2 + 3y^2 + 4z^2}$  and  $x = \sin t$ ,  $y = \cos t$  and  $z = \sin t$ . The final answer should be in terms of  $t$ . Show all of the work!
- Find  $w_t$ , in terms of  $s$  and  $t$ , given that  $w = \frac{x-z}{4y+z}$  and  $x = 4s + t$ ,  $y = 3st$ , and  $z = 4s - t$ .
- The volume of a right circular cylinder of radius  $r$  and height  $h$  is  $V = \pi r^2 h$  with  $r = e^{4t}$  and  $h = e^{-8t}$ . Find  $V'(t)$  and indicate if the volume of the cylinder increases or decreases as  $t$  increases.
- Assume the equation  $y \ln(x^4 + y^2 + 3) = 6$  implicitly defines  $y$  as a differentiable function of  $x$ , find  $\frac{dy}{dx}$ .
- Let  $f(x, y) = 2x^2 y$  where  $x = 2s + 2t$  and  $y = 2s - t$ . Find  $f_{st}$ .
- Find  $\nabla p(x, y)$  for the function  $p(x, y) = \sqrt{14 - 4x^2 - y^2}$ . Then find  $\nabla p(1, 1)$ .
- Find the directional derivative for  $f(x, y) = e^{-x-y}$  at the point  $P(\ln 3, \ln 2)$  in the direction of the vector  $\langle 1, 1 \rangle$ . Please note the magnitude of the given vector.
- Consider the function  $f(x, y) = 6 \sin(5x - 3y)$  and the point  $P(0, 2\pi)$ . Find the unit vectors that give the direction of steepest ascent and steepest descent at  $P(0, 2\pi)$ . Find a vector that points in a direction of no change in the function at  $P$ .
- Consider the paraboloid  $f(x, y) = 14 - \frac{x^2}{9} - \frac{y^2}{4}$  and the point  $P(3\sqrt{2}, -2)$  on the level curve  $f(x, y) = 11$ . Compute the slope of the tangent line to the level curve at  $P(3\sqrt{2}, -2)$ . Find the gradient of  $f(x, y) = 14 - \frac{x^2}{9} - \frac{y^2}{4}$  evaluated at  $P(3\sqrt{2}, -2)$ . What is the relationship between the tangent line and gradient if it is known that the slope of the tangent line can be used to write a vector in the direction of the tangent line,  $\langle 1, m \rangle$ ?
- Find the gradient of the function  $f(x, y, z) = e^{5xyz-3}$  at the point  $P(0, 4, 1)$  in the direction of the vector  $\mathbf{u} = \left\langle -\frac{6}{11}, \frac{7}{11}, -\frac{6}{11} \right\rangle$ . What is the directional derivative in the direction of  $\mathbf{u}$ ?

2. Let  $f(x, y, z) = e^{4x+4y-z}$ , which may be viewed as a level surface of the function  $w = f(x, y, z)$ . The point  $P(-2, 2, 0)$  is on the surface. The heads of all the vectors orthogonal to the gradient with their tails at  $P$  form a plane, find an equation of that plane.
3. Find an equation of the plane tangent to the surface  $z = 4\cos(x - y) + 3$  at  $\left(\frac{\pi}{6}, -\frac{\pi}{6}, 5\right)$ .
4. Find the equation for the linear approximation for the function  $f(x, y) = \sqrt{x^2 + y^2}$  at the point  $(-3, 4)$ . Then use this function to approximate  $f(-2.97, 4.06)$ .
5. Use differentials to approximate the change in  $z = \ln(x^{10}y)$  when  $(x, y)$  changes from  $(-5, 2)$  to  $(-4.97, 1.98)$ .
6. Find the differential  $dw$  for the function  $w = f(u, x, y, z) = \frac{5y + 7z}{x + u}$ .
7. Find all points on the surface  $z = x^2 - 4xy - y^2 - 6x + 2y$  where the tangent plane is horizontal.
8. Find all critical points for the function  $f(x, y) = 8xy - 2x^4 - 2y^4$ .
9. Find the critical points for the function  $f(x, y) = ye^x - 5e^y$  and then use the Second Derivative Test to determine if each critical point is a relative minimum, relative maximum, saddle point or state the test is inconclusive.
10. A rectangular metal tank with an open top is to hold 13.5 cubic feet of liquid. What are the dimensions of the tank that would minimize the materials needed to build the tank? Hint: you want to write the function to be minimized at a function of only two variables.
11. Find the absolute extrema of  $f(x, y) = x^2y - y^3$  on the region  $R = \{(x, y) | 0 \leq x \leq 2, 0 \leq y \leq 2 - x\}$
12. Use the technique of Lagrange multipliers to find the absolute extrema of  $f(x, y) = 2xy$  subject to the constraint  $2x^2 + 2y^2 - 3xy = 49$ .
13. Evaluate, give only exact answers:  $\int_4^9 \int_0^4 \sqrt{w} dw du$
14. Evaluate, give only exact answers:  $\int_0^2 \int_0^1 x^5 y^2 e^{x^3 y^3} dy dx$
15. Evaluate  $\iint_R e^{x+4y} dA$  for the region  $R = \{(x, y) | 0 \leq x \leq \ln 2, 1 \leq y \leq \ln 4\}$ .

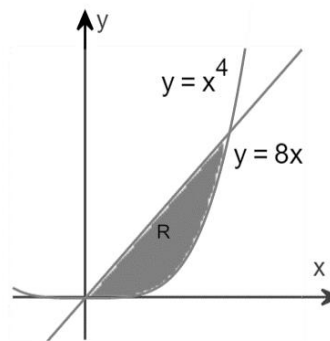
16. Write an iterated integral that can be used to find the volume of the solid between the cylinder  $f(x, y) = e^{-x}$  and the region  $R = \{(x, y) | 0 \leq x \leq \ln 5, -5 \leq y \leq 5\}$ .

17. Consider the double integral  $\iint_R x \sec^2(xy) dA$  with  $R = \{(x, y) | 0 \leq x \leq \frac{\pi}{6}, 0 \leq y \leq 2\}$ . Set up the iterated integral in the easiest order and evaluate.

18. Find the average value of the function  $f(x, y) = 6 \sin x \cos y$  over the region

$$R = \left\{ (x, y) \mid 0 \leq x \leq \frac{\pi}{3}, 0 \leq y \leq \frac{\pi}{2} \right\}.$$

19. Consider the region in the figure and write an iterated integral of a continuous function over  $R$ .



20. Evaluate, give only exact answers:  $\int_0^{\frac{3\pi}{2}} \int_y^{\frac{3\pi}{2}} 12 \sin(4x - 3y) dx dy$

21. Write an iterated integral of a continuous function  $f(x, y)$  over the region

$$R = \{(x, y) | 0 \leq x \leq y(12 - y)\}.$$

22. Reverse the order of integration for the integral  $\int_0^9 \int_{\frac{y}{3}}^{\sqrt{y}} f(x, y) dx dy$

23. Reverse the order of integration and then evaluate the integral  $\int_0^1 \int_y^1 3e^{x^2} dx dy$ . Why must the order be reversed?

24. Find the volume of the solid bounded by the coordinate planes and the plane  $4x + 2y + z - 8 = 0$ .

25. Use a double integral to find the area bounded by  $y = 12 + 12 \sin x$  and  $y = 12 - 12 \sin x$  on  $[0, \pi]$ .

26. Find the volume of the solid below the hyperboloid  $z = 8 - \sqrt{1 + x^2 + y^2}$  and above the region  $R = \{(r, \theta) \mid 2\sqrt{2} \leq r \leq 2\sqrt{6}, 0 \leq \theta \leq 2\pi\}$ .

27. Evaluate  $\iint_R 2xy dA$  using polar coordinates over the region  $R = \{(r, \theta) \mid 1 \leq r \leq 2, 0 \leq \theta \leq \frac{\pi}{2}\}$

28. Evaluate by converting to a polar integral  $\int_0^5 \int_0^{\sqrt{25-x^2}} \sqrt{x^2 + y^2} dy dx$ .

29. Express  $\iint_R f(r, \theta) dA$  as an iterated integral over the region outside the circle  $r = 2\sqrt{2}$  and inside the rose  $r = 4 \sin 2\theta$  in the first quadrant. No need to evaluate the integral.