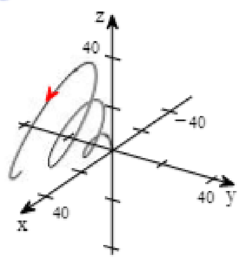
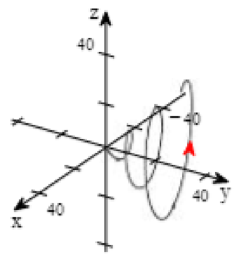
1. Which of the following graphs is the vector valued function $r(t) = -2t\cos t\mathbf{i} - 2t\mathbf{j} - 2\sin t\mathbf{k}$ for $0 \le t \le 6\pi$? Explain your reasoning.

○ A.

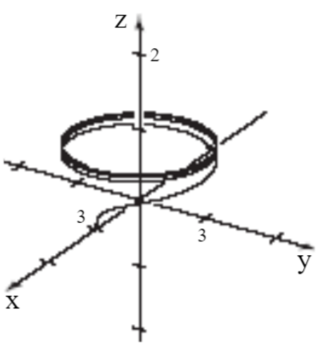


○ B.

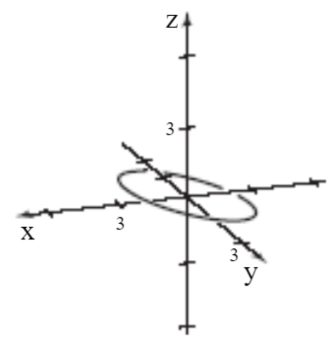


2. Which of the following graphs is the vector valued function $r(t) = 3\cos t^2\mathbf{i} + 3\sin t^2\mathbf{j} + \frac{t}{t+6}\mathbf{k}$ for $0 \le t \le \infty$? Explain your reasoning.

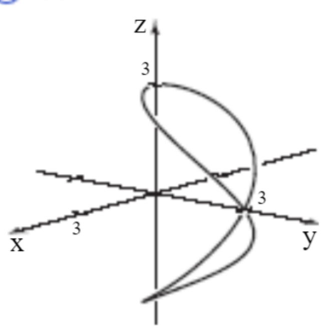
○ A.



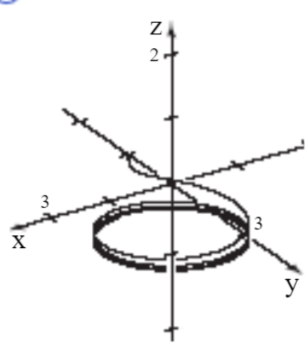
○ B.



○ C.



○ D.



3. Evaluate $\lim_{t\to\infty}\left(5e^{-t}\mathbf{i}-\frac{9t}{t+6}\mathbf{j}+3\tan^{-1}t\mathbf{k}\right)$.

4. Evaluate $\lim_{t\to 0}\left(\frac{\sin 3t}{3t}\mathbf{i}-\frac{e^{7t}-7t-1}{t}\mathbf{j}+\frac{3\cos t+\frac{3t^2}{2}-3}{t^2}\mathbf{k}\right)$.

5. Find the domain of the vector valued function $r(t)=\left\langle \sqrt{9-t^2},\sqrt{t},\frac{5}{\sqrt{4+t}}\right\rangle$.

6. Find the points, if they exist, where the plane $y-x=0$ intersects the curve $r(t)=\langle 7\cos t,7\sin t,t\rangle$ for $0\le t\le 4\pi$.

7. Find a tangent vector for $r(t)=\langle 7t,\cos 2t,5\sin t\rangle$ at $t=\frac{\pi}{2}$.

8. Find the unit tangent vector for $r(t)=\left\langle e^{2t},2e^{2t},2e^{8t}\right\rangle$ for $t\ge 0$.

9. Find the derivative of $\mathbf{u}(t)\bullet\mathbf{v}(t)$ where $\mathbf{u}(t)=8t^3\mathbf{i}+\left(t^2-6\right)\mathbf{j}-8\mathbf{k}$ and $\mathbf{v}(t)=e^t\mathbf{i}+7e^{-t}\mathbf{j}-e^{7t}\mathbf{k}$.

10. Suppose $\mathbf{u}$ is a differentiable function at $t=0$ with $\mathbf{u}(0)=\langle 0,1,-5\rangle$ and $\mathbf{u}'(0)=\langle 0,-6,3\rangle$. Use this information to evaluate $\frac{d}{dt}\left(\mathbf{u}(t)\cos t\right)\Big|_{t=0}$.

11. Let $\mathbf{v}(t)=\left\langle 5t^2,-6t,7\right\rangle$ and $g(t)=2\sqrt{t}$. Find the derivative of $\mathbf{v}(g(t))$.

12. Find $\int r(t)\,dt$ where $r(t)=\left\langle te^t,t^3\sin t^4,\frac{2t}{\sqrt{t^2+2}}\right\rangle$.

13. If it is known that $r'(t)=\langle e^t,-\sin t,\sec^2 t\rangle$ and $r(0)=\langle 2,2,2\rangle$, find $r(t)$.

14. Evaluate $\int_0^2 te^t(-3\mathbf{i}-5\mathbf{j}+\mathbf{k})dt$

15. Suppose the vector valued function $r(t)=\langle f(t),g(t),h(t)\rangle$ is smooth on an interval containing $t=t_0$. The line tangent to the curve at $t=t_0$ is the line parallel to the vector $r'(t_0)$ that passes through $(f(t_0),g(t_0),h(t_0))$. Use this to find the line tangent to $r(t)=\langle 10+\cos t,2+\sin 8t,3t\rangle$ at $t=\dfrac{\pi}{2}$.

16. Consider the helix $r(t)=\langle \cos t,\sin t,7t\rangle$ for $-\infty\le t\le\infty$. Find all points on the helix where $\mathbf{r}$ and $\mathbf{r}'$ are orthogonal.

17. Find the velocity vector $\mathbf{v}(t)$ for $r(t)=\left\langle 3\sin t+\sqrt{7}\cos t,\sqrt{7}\sin t-3\cos t\right\rangle$ for $0\le t\le 2\pi.$ Then find $\mathbf{r}\bullet\mathbf{v}$, explain why this dot product is equal to 0.

18. Given the acceleration vector $\mathbf{a}(t)=\langle 5t,e^{-t},3\rangle$ and initial velocity $\mathbf{v}_0=\langle 0,0,1\rangle$ and initial position vector $\mathbf{r}_0=\langle 4,0,0\rangle$ find both the velocity vector and position vector.

19. A small rocket is fired from a launch pad 15 m above the ground with an initial velocity $\langle 300,450,550\rangle m/s$. A cross wind blowing to the north produces an acceleration of the rocket of $3m/s^2$. Assume the x axis points east, the y axis points north, the positive z axis is vertical (opposite $g=-9.8m/s^2$), and the ground is horizontal. Find the following:
    a) velocity vector for $t\ge 0$
    b) position vector for $t\ge 0$
    c) time of flight
    d) range of the rocket
    e) maximum height of the rocket

20. A projectile is fired over horizontal ground from the origin with an initial speed of 80 m/s. What firing angle, in degrees, will produce a range of 250m?

21. Find the length of the curve $r(t)=\langle 9\cos t+9t\sin t, 9\sin t-9t\cos t\rangle$ where $0\le t\le \frac{\pi}{2}$.

22. For the trajectory $r(t)=\langle 4e^t\sin t, 4e^t\cos t, 4e^t\rangle$ where $0\le t\le \ln 2$, find the speed of the trajectory and the length of the trajectory on the given interval.

23. Given $r(t)=\langle \sqrt{195}\cos t, \cos t, 14\sin t\rangle$ find the unit tangent vector and the curvature for the curve.

24. Given $r(t)=\langle 5t, 5\ln(\cos t)\rangle$ for $-\frac{\pi}{2}<t<\frac{\pi}{2}$, find both the unit tangent vector and the principle normal vector.

25. Find both the tangential and normal components of the acceleration for $r(t)=\langle 18\cos t, 18\sin t, 26t\rangle$.

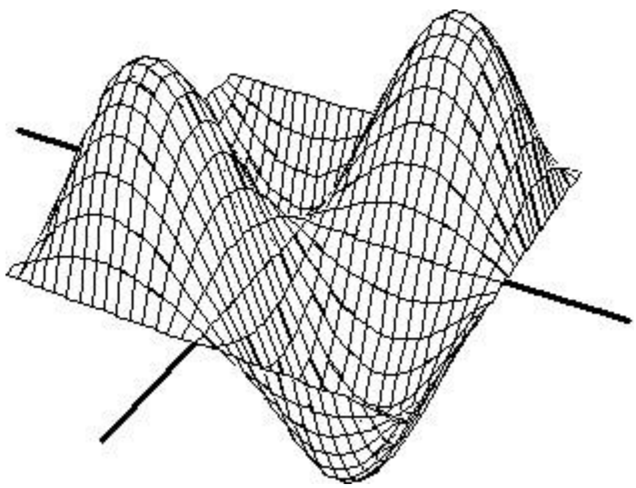
26. The parameterized curve $r(t)=\langle 15\sin t, 15\cos t, 16t\rangle$ has a unit tangent vector of $\frac{\langle 15\cos t, -15\sin t, 16\rangle}{\sqrt{481}}$ and unit normal vector $\langle -\sin t, -\cos t, 0\rangle$. Find both the unit binormal vector and torsion for $r(t)$.

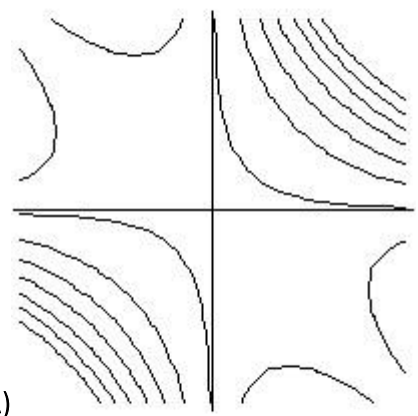
27. Find the domain of the following functions:

a) $f(x,y)=\sqrt{5-5x^2-5y^2}$

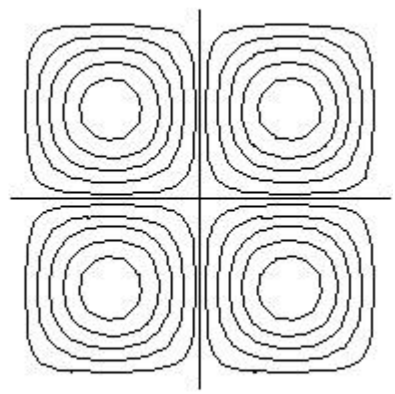
b) $f(x,y)=\sin^{-1}(2y-4x^2)$

28. Match the surface to its level curve:

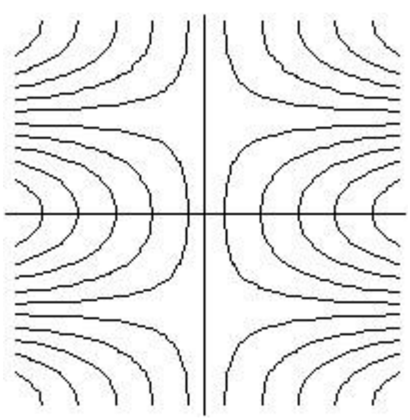




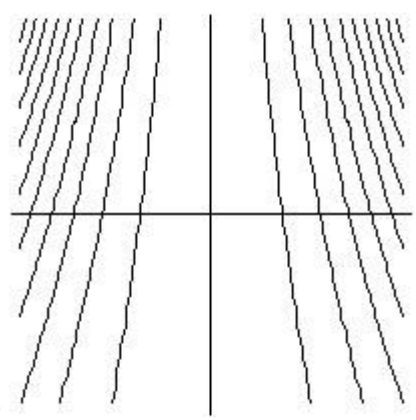
A)



B)

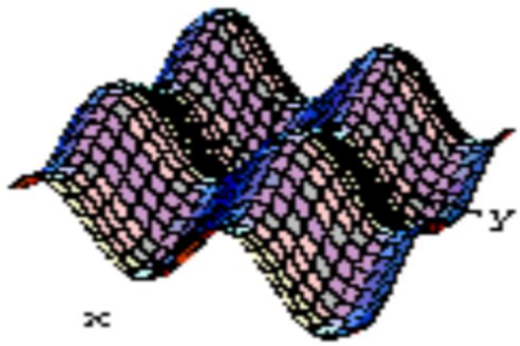


C)

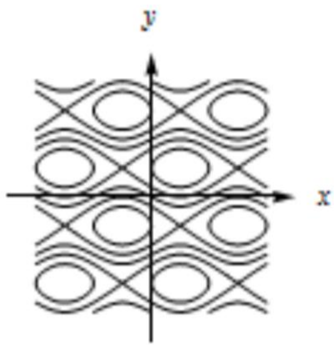


D)

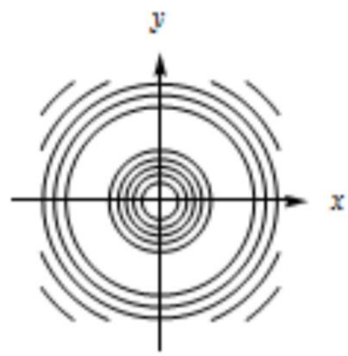
29.



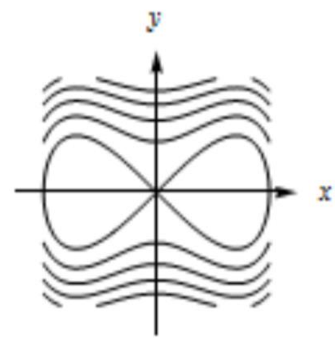
C.



A.



B.



30. Evaluate the following limit: $\lim_{(x,y)\to(4,5)} \dfrac{\sqrt{x+y}-3}{x+y-9}$.

31. Prove that the following limit DNE: $\lim_{(x,y)\to(0,0)} \dfrac{x+2y}{x-2y}$.

32. Find any points of discontinuity for

$$f(x,y)=\begin{cases} \dfrac{\sin\left(6x^2+5y^2\right)}{6x^2+5y^2} & if\ (x,y)\neq(0,0) \\ 1 & if\ (x,y)=(0,0) \end{cases}$$

33. Find both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y)=10e^{x^2 y}$.

34. Find both f_x and f_y for $f(x,y)=\cos^5\left(x^9 y^5\right)$

35. Find the four second partial derivatives for $z=7ye^{9x}$.

36. Find both f_{xy} and f_{yx} for $f(x,y)=9x^6y^3-7x^5y^4$.

Formulas for Curves in Space

$$\mathbf{T}=\frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N}=\frac{\frac{d\mathbf{T}}{dt}}{\left|\frac{d\mathbf{T}}{dt}\right|}$$

$$\kappa=\left|\frac{d\mathbf{T}}{ds}\right|=\frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right|=\frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$$

$$\mathbf{a}=a_N\mathbf{N}+a_T\mathbf{T} \text{ where } a_N=\kappa|\mathbf{v}|^2=\frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|} \text{ and } a_T=\frac{d^2s}{dt^2}=\frac{\mathbf{v}\bullet\mathbf{a}}{|\mathbf{v}|}$$

$$\mathbf{B}=\mathbf{T}\times\mathbf{N}=\frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

$$\tau=-\frac{d\mathbf{B}}{ds}\bullet\mathbf{N}=\frac{(\mathbf{v}\times\mathbf{a})\bullet\mathbf{a}'}{|\mathbf{v}\times\mathbf{a}|^2}$$