

Evaluate the following indefinite integrals. Leave no negative exponents or compound fractions.

1.  $\int \sqrt[3]{x^2} \left( \frac{1}{x} - x^3 \right) dx =$

2.  $\int \frac{\cos^3 x}{2 - 2\sin^2 x} dx =$

3.  $\int \left( \frac{2x-3}{x} - e^{2x} \right) dx =$

4.  $\int \frac{e^x}{1+e^x} dx =$

5.  $\int \frac{1}{x^2 e^{\frac{5}{x}}} dx =$

6.  $\int 3 \tan^3 4x \sec^2 4x dx =$

7.  $\int \frac{x}{\sqrt{x-1}} dx =$

8.  $\int \frac{(\ln x)^5}{x} dx =$

9.  $\int \frac{8x^2 + 9x + 8}{1+x^2} dx =$

10.  $\int \left( \sin^2 \frac{x}{3} + \cos^2 \frac{x}{3} \right) dx =$

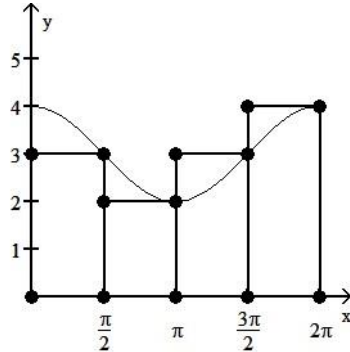
11.  $\int \frac{dx}{\sqrt{8+2x-x^2}} =$

12. Find  $f(x)$  if  $f''(x) = x^2$ ,  $f'(0) = 7$  and  $f(0) = 2$ .

13. Use a mid-point Riemann sum to approximate the area bounded by  $f(x) = \sqrt{7+x} + 1$  on the interval  $[0, 9]$  with  $n = 4$  subintervals.

14. Write the Riemann Sum  $\lim_{\|\Delta\| \rightarrow 0} \sum_{k=1}^n \left( 4(x_k)^2 - 6x_k + 7 \right) \Delta x_k$   $[-4, 2]$  as a definite integral.

15. Approximate the area under the curve  $f(x) = \frac{1}{x}$  above the  $x$  axis between  $x = 3$  and  $x = 9$  using the midpoint as  $x_k$  and two rectangles of equal width.
16. The following graph shows how a student decided to approximate the area under the curve. How many rectangles were used? What interval were they integrating over? How were they defining the height?



Evaluate the following definite integrals using the Fundamental Theorem of calculus. Give exact answers. Do NOT give decimal approximations!! If decimal approximations are given NO credit will be awarded.

17.  $\int_0^{\pi} \sin \frac{x}{2} \cos \frac{x}{2} dx =$

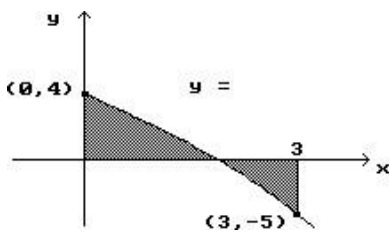
18.  $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{3}{\sqrt{4-9x^2}} dx =$

19.  $\int_0^{\ln 3} \frac{e^x}{\sqrt{e^x + 1}} dx =$

20.  $\int_2^3 \frac{x^3}{x^4 - 3} dx =$

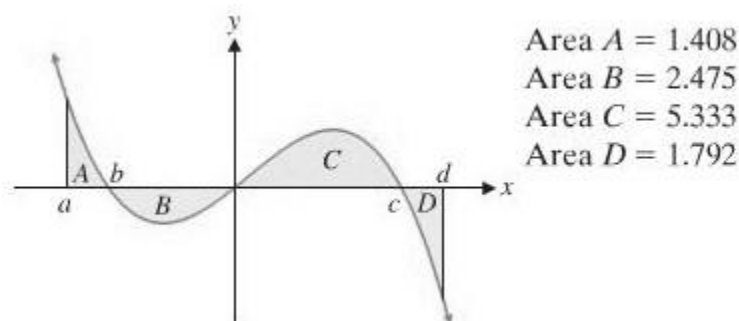
21. Find the area bounded by  $y = x^2 - 4$ ,  $x = -3$ ,  $x = 3$  and the  $x$  axis.

22. Given the graph of a function to be where  $(2,0)$  is the  $x$  intercept shown.



Would you expect  $\int_0^3 f(x) dx$  to be positive or negative. Explain your reasoning.

23. Use the following graph to find the indicated value.



a)  $\int_a^c f(x)dx$       b)  $\int_c^d f(x)dx$       c)  $\int_0^a f(x)dx$

24. TRUE or FALSE: The average value of  $f(x) = x^3$  over the interval  $[-3, 3]$  is  $\frac{27}{4}$ . Explain.

25. Find the  $c$  guaranteed by the Mean Value Theorem for Integrals with  $f(x) = \sqrt{x+1}$  on the interval  $[0, 15]$

26. TRUE or FALSE:  $\int x^2 e^x dx = \left(\frac{1}{3} x^3 e^x\right) + C$ . Explain.

27. Given  $\int_0^9 f(x)dx = 5$  and  $\int_3^9 f(x)dx = -1$ , find

a)  $\int_0^3 f(x)dx$    b)  $\int_9^0 f(x)dx$    c)  $\int_3^3 f(x)dx$    d)  $\int_{-9}^9 f(x)dx$  if  $f(x)$  is even   e)  $\int_{-9}^9 f(x)dx$  if  $f(x)$  is odd

28. Use this information to find all of the following.  $\int_1^4 f(x)dx = 6$ ,  $\int_1^4 g(x)dx = 4$  and  $\int_3^4 f(x)dx = 2$

a)  $-\int_4^1 3f(x)dx$

b)  $\int_4^1 (3f(x) - 2g(x))dx$

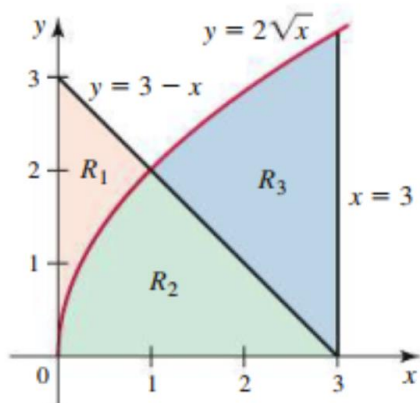
c)  $\int_1^4 \frac{f(x)}{g(x)}dx$

29. Use the definition of the definite integral with right Riemann sums and a regular partition to evaluate

$$\int_1^2 (3x^2 + x)dx$$

30. Find the area in the first quadrant bounded by the curve  $\sqrt{x} + \sqrt{y} = 1$ .

31. Find the area of  $R_2$  by setting up a single integral.



32. Find the following limits:

a)  $\lim_{x \rightarrow 0} \frac{3 \sin^2 2x}{x^2}$

b)  $\lim_{x \rightarrow 0} \frac{e^{-2x} - 1 + 2x}{x^2}$

c)  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 + x + 1} - \sqrt{x^2 - x} \right)$

d)  $\lim_{x \rightarrow \infty} \ln \left( \frac{x+1}{x-1} \right)$

e)  $\lim_{x \rightarrow 1} (x-1)^{\sin \pi x}$

f)  $\lim_{x \rightarrow \infty} \left( \frac{2}{\pi} \tan^{-1} x \right)^x$