MTH-140 Review for Test 3 Luczak

1. Find $\frac{dy}{dx}$ for each of the following functions.

   a. $f(x)=\sin^{-1}(\cos x)$

   b. $f(x)=x^2\pi^x$

   c. $f(x)=(x+1)^{e^{2x}}$

   d. $f(x)=10^{\csc 3x}$

   e. $f(x)=\log_5(x^2-2x+1)$

   f. $y=e^{\sin^{-1}(8x)}$

   g. $y=\sqrt{\cos^{-1}9x}$

   h. $\cos(xy)=3$

2. If $f(x)=3-|x-4|$, then $f(1)=f(7)$ but $f'(x)\neq 0$ for any $x\in[1,7]$. Does this contradict Rolle's Theorem? Why or why not?

3. Verify that the Mean Value Theorem applies to the function $f(x)=\sqrt{10x}$ on the interval $[0,10]$ an then find the value c such that $\frac{f(b)-f(a)}{b-a}=f'(c)$.

4. Let $f(x)=xe^x$

   a. Find all critical numbers (x-values)

   b. Use the second derivative test to determine the relative extrema. Label the extrema as relative maximum or relative minimum. (Note: extrema are ordered pairs.)

5. Given $f(x)=\sin x\cos x$ find:

   a. absolute max on $[0,\pi]$ (ordered pair(s))

   b. absolute min on $[0,\pi]$ (ordered pair(s))

6. Find a function that has vertical asymptotes at $x=\pm 1,0$ and a horizontal asymptote at $y=\frac{2}{3}$.

7. Let $f(x)=\dfrac{3x^3+11x^2+11x+2}{x^2-4}$. Find all asymptotes.

8. Consider the statement: If $f(x)$ is a function that is differentiable everywhere and $f'(3)=0$, then $(3, f(3))$ is relative extrema of $f(x)$. Is this statement true or false. **Explain your answer**.

9. Find the relative extrema for the function $f(x)=x^{\frac{2}{3}}(x-2)^2$.

10. Given $f(x)=x^4-4x^3+10$

    a. Find $f'(x)=$ _____

    b. list all critical numbers _____

    c. Use the first derivative test to determine if the critical number(s) give relative extrema. Give the ordered pair and label.

    d. on which intervals is $f(x)$ increasing? _____

    e. on which intervals is $f(x)$ decreasing? _____

    f. Find $f''(x)=$ _____

    g. on which intervals is $f(x)$ concave up? _____

    h. on which intervals is $f(x)$ concave down? _____

    i. inflection points (ordered pair)

11. Sketch the graph that yields the following:

$f(0)=4,\ f(3)=1,\ \text{and}\ f(4)=3$

$f'(0)=\ f'(4)=0$

$f'(3)$ DNE

$f'(x)>0$ on $(3,4)$

$f'(x)<0$ on $(-\infty,0),\ (0,3),\ \text{and}\ (4,\infty)$

$f''(0)=0$

$f''(3)$ DNE

$f''(x)>0$ on $(-\infty,0)$

$f''(x)<0$ on $(0,3)$ and $(3,\infty)$

12. Let $f''(x)=3x^2-9$ and let $f(x)$ have critical numbers $x=-3,0,3$. Use the second derivative test to determine which critical numbers if any give relative extrema.

13. Gravitational force is inversely proportional to the distance between two objects squared. If $F = \left(\frac{54}{d^2}\right)$ where F is the gravitational force and d is the distance. How fast is the force diminishing at the instant the objects are 3 meters apart and moving at 2.2 m/s?
14. Calculate Δy and dy for $f(x) = \frac{1}{x^2}$ when $x = 2$ and $\Delta x = -.1$. (Round answers to 5 decimal places where needed.)
15. The management of a large store has 1600 feet of fencing to fence in a rectangular storage yard using the building as one side of the yard. If the fencing is used for the remaining 3 sides, find the dimensions that will give maximize the area of the yard. What is the area?
16. A poster is to contain 96 in^2 of printed matter with margins of 4 inches each at top and bottom and 3 inches each to the left and right. Find the dimensions of the printed portion if the total area of the poster is to be a minimum.
17. Prove one of the following:
    a. $\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{u'}{\sqrt{1-u^2}}$
    b. $\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{u'}{1+u^2}$
    c. $\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{u'}{|u|\sqrt{u^2-1}}$