Calculus I
Review for Test 1
Luczak

SHOW ALL WORK. If no, or insufficient work is shown you will not receive credit. All answers must be in exact form (no decimal approximations).

1. Find $\lim_{x\to 4}\left(\frac{3}{4}x+5\right)=$ ________________

   Prove the above limit. [Hint: find δ first.]

2. Let $f(x)=\frac{x-7}{x^2-49}$. Is $f(x)$ continuous at $x=7$? If your answer is yes show how $f(x)$ meets all three conditions to be continuous at a point. If your answer is no show how the three conditions are not all met.

3. Let $f(x)=\frac{3x}{x^4+1}$. Is $f(x)$ continuous at $x=-1$? If your answer is yes show how $f(x)$ meets all three conditions to be continuous at a point. If your answer is no show how the three conditions are not all met.

4. Sketch a single function, $f(x)$, that meets the following criteria:

   a) $f(x)$ has a jump discontinuity at $x=-2$.
   b) $f(x)$ has a removable discontinuity at $x=1$.
   c) $f(x)$ has an infinite discontinuity at $x=3$.
   d) has only the above points of discontinuity.

5. Sketch a single function, $f(x)$, that meets the following criteria:

   $\lim_{x\to\infty} f(x)=3$ $\lim_{x\to-\infty} f(x)=-3$

   $\lim_{x\to 1^+} f(x)=\infty$ $\lim_{x\to 1^-} f(x)=\infty$

6. Let $f(x)=\begin{cases}\sin 2x & \text{if } x\geq 0\\ x^3-x+1 & \text{if } x<0\end{cases}$

   a) list the points of discontinuity (if any)________________________________

   b) classify the points of discontinuity as removable, jump, or infinite___________

7. Let $f(x)=\dfrac{\frac{1}{x+4}-\frac{1}{4}}{x}$.

a) list the points of discontinuity (if any)________________________________

b) classify the points of discontinuity as removable, jump, or infinite__________

8. Find the following limits. To receive credit you must show algebraic work. Use $\pm\infty$ where appropriate.

a) $\lim\limits_{x\to 0}\dfrac{1-e^{-x}}{1+e^{x}}=$

b) $\lim\limits_{h\to 0}\dfrac{3(x+h)^2-4(x+h)+1-\left(3x^2-4x+1\right)}{h}=$

c) $\lim\limits_{x\to 4^-}\dfrac{\sqrt{x}-2}{x-4}=$

d) $\lim\limits_{x\to 1}\dfrac{1-x}{x^2+2x-3}=$

9. Find the following limits by any means. Use $\pm\infty$ where appropriate.

a) $\lim\limits_{x\to\pi}x\sec x=$

b) $\lim\limits_{x\to 0}e^{\frac{\sin x}{x}}=$

c) $\lim\limits_{x\to 1^+}\dfrac{x^2+2x+1}{x-1}=$

d) $\lim\limits_{x\to\infty}\dfrac{5x^2-10x+6}{5x^3-2x+4}=$

e) $\lim\limits_{x\to 0}\dfrac{x}{\sin 3x}=$

f) $\lim\limits_{x\to 0^+}\ln x=$

g) $\lim_{x\to\infty} \frac{3x}{\sqrt{x^2+3}} =$

10. Determine whether the following statements are true or false. If it is true explain why it is true. If it is false, give an example to show that it is false.

a) A rational function has at least one vertical asymptote.

b) If $f(x) = g(x)$ for all real numbers other than $x = 0$, and $\lim_{x\to 0} f(x) = L$, then $\lim_{x\to 0} g(x) = L$.

11. Suppose $f(x)$ is continuous on the interval $(0,2)$ and never equal to zero there, and that $f(1) > 0.$ What can you say, if anything, about the sign of $f(x)$ on the interval $(0,2)$?

BONUS: Determine values of c and d so that the function is continuous everywhere.

$$f(x) = \begin{cases} 4x & \text{if } x < -1 \\ cx + d & \text{if } -1 \le x \le 2 \\ -5x & \text{if } x > 2 \end{cases}$$